A Method for Forward Displacement Analysis of 3-RRP and 3-PRP Planar Parallel Manipulators

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Abstract: The paper presents a method for the forward displacement analysis of 3-RRP and 3-PRP planar parallel manipulators including the RP-RP-RP third-class Assur group. The aim of this analysis is to find all possible configurations of the parallel planar mechanism for one given set of input joints values. The proposed method leads to a non-linear system of three equations with three unknown parameters. Using a successive elimination procedure, a polynomial equation of eighth order in one unknown is obtained. The real solutions of the polynomial equation correspond to the assembly modes of the planar parallel mechanism. The maximum number of the assembly modes of the investigated manipulators is two. Furthermore, a numerical application of the proposed method is presented.

Keywords: planar parallel manipulator, forward displacement analysis, Assur group.

1. INTRODUCTION

Parallel manipulators have some potential advantages over serial manipulators such as higher stiffness and better accuracy, higher velocities and accelerations, greater payload-to-weight ratio, possibility to locate actuators on the fixed base. However, they present limited workspace and multiple direct kinematic solutions (Tsai, 1999).

In the parallel planar manipulators all the moving links perform planar motions. Different methods for the direct kinematics of 3-DOF planar parallel manipulators have been developed by Gosselin et al. (1992), Mohammadi-Daniali et al. (1993), Merlet (1996), and Kong and Gosselin (2001).

This paper presents a method for the forward displacement analysis of the 3-RRP and 3-PRP planar parallel manipulators, where R, P, R, P denote revolute, prismatic, actuated revolute and actuated prismatic joints, respectively. The developed method takes into account that the 3-DOF planar parallel manipulators under the study are multi-loop mechanisms with decoupled structure, where the input links are connected with the fixed base and with one third-class Assur group (Mitsi et al., 2003), (Mitsi et al., 2008).

The paper is organized as follow. Section II describes the procedure for the direct kinematics solution of planar parallel manipulators under the study. Section III presents a methodology for displacement analysis of the RP-RP-RP third-class Assur group. Finally, a numerical application and conclusions are given.

2. PROCEDURE FOR FORWARD DISPLACEMENT ANALYSIS OF THE MANIPULATORS

The 3-RRP and 3-PRP planar parallel manipulators (Fig. 1) are closed-loop mechanisms with three degrees of freedom, composed of three RRP and PRP planar kinematical chains respectively with identically topology, all connecting the fixed base $O_1O_2O_3$ to the moving platform $P_1P_2P_3$. The actuated joints are located on the fixed base and are revolute R and prismatic P, respectively. It is observed that the both planar parallel manipulators contain, except the fixed link and input links, a RP-RP-RP third-class Assur group.

The forward displacement analysis of the manipulators under the study may be formulated as follows: given the geometry of the links, the coordinates of the fixed base points O_1, O_2 and O_3 and the set of input joints values (angles φ_{i0} and displacements s_{i0} (i=1,2,3) in Fig. 1) find the pose of the moving platform $P_1P_2P_3$.

The above mentioned planar parallel manipulators have a decoupled structure (Mitsi et al., 2003), where the external joints of the Assur group are connected with the known moving links (here input links). For the forward displacement analysis is used a modular method, where the constraint equations can be written and solved separately for each module, in hierarchical order defined by the mechanism structure. The forward displacement analysis procedure of the 3-RRP and 3-PRP planar parallel manipulators consists of the following steps:

Determination of the joints coordinates A_1, A_2, and A_3 with respect to the frame coordinate system of the mechanism:

$$r_{Ai} = [x_{Ai} \quad y_{Ai}]' = [x_{Oi} + l_{OiAi} \cos\varphi_{i0} \quad y_{Oi} + l_{OiAi} \sin\varphi_{i0}]' \tag{1}$$

for 3-RRP manipulator

$$r_{Ai} = [x_{Ai} \quad y_{Ai}]' = [x_{Oi} + s_{0i} \cos\varphi_{i0} \quad y_{Oi} + s_{0i} \sin\varphi_{i0}]' \tag{2}$$

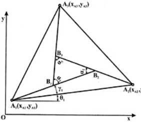

Fig. 3. Kinematic model of the RP-RP-RP Assur group

The system of nonlinear equations (9)-(11) is solved through successive application of the resultant method (Van der Waerden, 1991). Since the variable s_2 is present only in (9) and (10), using the Sylvester theorem, one can eliminate s_2 from these equations yielding:

$$F_{12}(s_1,s_3)=0 \tag{12}$$

Again, using Sylvester theorem and eliminating the variable s_3 from (11) and (12) yields the eighth order polynomial equation in the only unknown s_1:

$$\sum_{i=0}^{8} P_i s_1^i = 0 \tag{13}$$

The coefficients P_i (i=0,1,...8) depend only on the Assur group data. Equation (13) provides eight roots for s_1 in the complex field from which maximum two real solutions lead to real solutions for displacements s_2 and s_3 (Pelecudi, 1975),(Chung, 2005). Therefore, the maximum number of the assembly modes of the RP-RP-RP Assur group with three external revolute joints and three internal prismatic joints is two. Furthermore, the maximum number of the assembly modes of the 3-RRP and 3-PRP planar parallel manipulators is two.

For every real solution for displacement s_1, the coordinates of the points B_1, B_2, and B_3 are determined.

The coordinates of the point B_1 are (see Fig. 3):

$$r_{B1}=\begin{bmatrix} x_{B1} & y_{B1}\end{bmatrix}^T = \begin{bmatrix} x_{A1}+s_1\cos\varphi_1 & y_{A1}+s_1\sin\varphi_1\end{bmatrix}^T \tag{14}$$

where: $\varphi_1=\theta_1+\gamma_1$ (15)

with θ_1= atan $[(y_{A2}-y_{A1})/(x_{A2}-x_{A1})]$ and angle γ_1 evaluated by considering the law of sines for triangle $A_1B_2A_2$:

$$\sin\gamma_1 = s_2 \sin\alpha_2 / l_{A1A2} \tag{16}$$

Furthermore, the coordinates of the points B_2 and B_3 are respectively:

$$r_{B2}=\begin{bmatrix} x_{B2} & y_{B2}\end{bmatrix}^T = \begin{bmatrix} x_{B1}+l_1\cos\varphi_1 & y_{B1}+l_1\sin\varphi_1\end{bmatrix}^T \tag{17}$$

$$r_{B3}=\begin{bmatrix} x_{B3} & y_{B3}\end{bmatrix}^T = \begin{bmatrix} x_{B1}+l_3\cos(\varphi_1+\alpha_1) & y_{B1}+l_3\sin(\varphi_1+\alpha_1)\end{bmatrix}^T \tag{18}$$

4. NUMERICAL APPLICATION

In this section the proposed procedure is applied to a symmetrical planar 3-RRP parallel manipulators, where $O_1O_2=O_2O_3=O_3O_1$, $O_1A_1=O_2A_2=O_3A_3$, $l_1=l_2=l_3$, and $\alpha_1=\alpha_2=\alpha_3$ (see Fig.1). The geometrical data and the input links position angle φ_{i0} (i=1,2,3) are given in the Table 1. For the specific geometry here considered, by solving the polynomial equation (13), two real solutions of the displacement s_1 are obtained. For each real solutions of the s_1 (see Table 1), the displacements s_2, s_3, and the coordinates of the points B_1, B_2, and B_3 are calculated. The pose and orientation of the moving platform is defined using the coordinates of the points B_1 and B_2 given in the Table 2. The corresponding two assembly modes of the planar 3-RRP parallel manipulator are illustrated in Fig. 4.

Table 1. Data and solutions of the 3-RRP parallel manipulator

Data	$x_{A1}=0$, $y_{A1}=0$, $x_{A2}=140$, $y_{A2}=0$, $x_{A3}=70$, $y_{A3}=121.244$, $\varphi_{10}=60^0$, $\varphi_{20}=100^0$, $\varphi_{30}=210^0$, $l_{OA1}=l_{OA2}=l_{OA3}=20$, $l_1=l_2=l_3=40$, $\alpha_1=\alpha_2=\alpha_3=60^0$		
Config.	s_1	s_2	s_3
1	-77.5558	-103.5205	-79.5285
2	42.7405	62.9381	34.9262

Table 2. Coordinates of the moving platform points

Config.	Coordinates of moving platform points B_1, B_2, and B_3		
	x_{B1} y_{B1}	x_{B2} y_{B2}	x_{B3} y_{B3}
1	63.6963 73.2813	36.0020 44.4191	74.8446 34.8662
2	48.2205 36.4501	83.9904 54.3532	50.6010 76.3792

5. CONCLUSIONS

Compared with other methods, the proposed procedure solves simultaneously the forward kinematics analysis of two planar parallel manipulators 3-RRP and 3-PRP. Furthermore, the procedure can be extended for direct kinematics analysis of 3-DOF planar parallel manipulators with actuated joints located on the fixed base as 3-RRR, 3-PRR including a RR-

RR-RR Assur group and 3-RPR, 3-PPR including a PR-PR-PR Assur group.

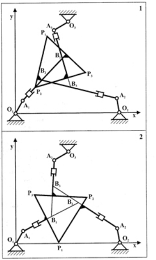

Fig. 4. Assembly modes of the 3-RRP parallel manipulator

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Appendix A.

The coefficients a_{ij} (i=1,2,3 and j=1,2,3,4) of (9) - (11) are:

$$a_{11} = 2\cos\alpha_2$$
$$a_{12} = 2l_1$$
$$a_{13} = 2l_1\cos\alpha_2$$
$$a_{14} = l_1^2 - l_{B1B2}^2$$

$$a_{21} = 2\cos\alpha_3$$
$$a_{22} = 2l_2$$
$$a_{23} = 2l_2\cos\alpha_3$$
$$a_{24} = l_2^2 - l_{B2B3}^2$$

$$a_{31} = 2\cos\alpha_1$$
$$a_{32} = 2l_3$$
$$a_{33} = 2l_3\cos\alpha_1$$
$$a_{34} = l_3^2 - l_{B3B1}^2$$